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Evaluation of Shear-Induced Particle Diffusivity in Red Cell Ghosts Suspensions

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Abstract−The shear-induced particle diffusivity in the red blood cell suspensions was evaluated based on the flow model and experimental results in a rectangular flow chamber. The effective diffusivity (D_e) of solute in the particle suspensions is equal to the stationary diffusivity (D_s) of the solute plus the shear-induced particle diffusivity (D_p). The effective diffusivity (D_e) of bovine serum albumin (BSA) in the red blood cell (RBC) ghost suspensions was determined under diffusion-limited conditions using a total internal reflection fluorescence (TIRF) method as a function of suspended RBC ghost volume fractions (0.05-0.7) and shear rates (200-1,000 s^{-1}). The stationary diffusivity (D_s) of BSA in RBC ghost suspensions was calculated by Meredith and Tobias model. Therefore the shear-induced particle diffusivity undergoing laminar shear flow can be evaluated. The shear-induced RBC ghost diffusivity was ranged from 0.35×10^{-7} to 21.2×10^{-7} cm^2/s and it increased with increasing shear rate. Also the shear-induced RBC ghost diffusivity increased as a particle volume fraction increased as well, up to a particle volume fraction of 0.45. However, for RBC ghost volume fractions above 0.45, the shear-induced particle diffusivity decreased with increasing particle volume fraction. The shear-induced particle diffusivity in RBC ghost suspensions is a function of a particle Peclet number (or shear rate) and particle volume fractions. The dimensionless particle diffusivity ($D_p/a^2\gamma$) was investigated as a function of particle volume fraction and these results are in good agreement with the literature values.

Key words: Shear-Induced Particle Diffusivity, Effective Diffusivity, Suspension, Red Blood Cell Ghost, Particle Motion

INTRODUCTION

Suspensions of small particles in fluids are common both in nature and in several engineering fields [Eckstein et al., 1977; Yim, 1999; Chin and Park, 2001]. A well-known example is blood, which contains mainly red blood cells, platelets and white blood cells in plasma. Shear-induced particle diffusion in blood is important in at least the following areas [Eckstein et al., 1977]: (i) In extracorporeal devices where red blood cells are hemolyzed when they come into contact with artificial surfaces. (ii) In the formation of clots, or thrombi, where part is determined by the diffusional fluxes of platelets to the arterial walls. (iii) In oxygen transport where it is largely transported by attachment to the hemoglobin of red blood cells and in the movement of red blood cells, the shear-induced particle diffusion produces augmented diffusion.

For particles that are suspended in a fluid, two fundamentally different approaches - the macroscopic or bulk properties of the suspension and the microscopic structure on the particle scale - exist for development of constitutive relations. The macroscopic approach is based on continuum mechanics, in which the detailed constitutive model is derived from an initial assumption of the appropriate functional form between dependent and independent variables. Having postulated a constitutive form without reference to any specific material the theory has no way of predicting any of the material parameters of the model. On the other hand, the microscopic structural approach begins with a physical description of a specific material, and then attempts to deduce its macroscopic properties by analyzing its behavior at the microscale, and then passing to the macroscale by an appropriate averaging process. For suspensions, this averaging process is, in general, a statistical one owing to the random distributions of particle motion, position, orientation, shape, and size which characterize the microscopic structural state [Leal, 1977].

The coefficient of shear-induced particle self-diffusion in concentrated suspensions of solid sphere was evaluated using a random walk model from measurements of transit times of a labeled particle in a Couette flow device [Leighton and Acrivos, 1987]. Eckstein et al. [1977] experimentally studied the self-diffusion of particles in the shear flow of a suspension by tracking the lateral motion of individual radioactively-labeled polystyrene particles in a Couette flow device. The effective particle diffusion coefficient was evaluated by means of random walk theory, using the ergodic hypothesis. Zydney and Colton [1988] proposed that augmented solute transport in the shear flow of concentrated suspensions arose primarily from shear-induced particle motion. The augmentation in solute transport is thus closely linked to the shear-induced particle diffusivity. They found the relationship that the effective solute diffusivity in the suspension was given by the sum of the stationary flow and particle diffusivity.

In this study, the shear-induced particle diffusivity (D_p) was evaluated by the relation $D_e=D_s+D_p$ where D_e is the effective diffusivity of solute in the suspension, D_s is the diffusivity in stationary suspension [Zydney and Colton, 1988]. This relation comes from applying the microscopic structure approach to the convection/diffusion model and continuity equation. The effective diffusivity of bovine serum albumin (BSA) in the RBC ghost suspensions was measured using total internal reflection fluorescence method in the rec-

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tangular flow chamber at a shear rate range of 200-1,000 s^{-1} and RBC ghost volume fraction range of 0.05 to 0.70 [Cha and Beissinger, 1996b]. The BSA transport experiment to measure the effective diffusivity was done under diffusion-limited conditions in the rectangular flow chamber system. Experimental data were analyzed by the Leveque solution to get the effective diffusivity of BSA. The diffusivity in a stationary suspension was evaluated by the Meredith and Tobias model [Meredith and Tobias, 1968]. Also, the shear-induced diffusivity of the RBC ghost particles was correlated by the dimensionless particle diffusivity empirically by

$$\frac{D_p}{a^2\gamma} = m\phi_p(1-\phi_p)^n$$

where m and n are constant, a is the radius of RBC ghost particle, γ is the shear rate, and ϕ_p is the ghost volume fraction.

FLOW MODEL FOR SHEAR-INDUCED PARTICLE TRANSPORT

In two-phase flow, where a solute (in this study BSA) is dissolved in a continuous phase (in this study phosphate buffered saline (PBS) solution) which contains a discontinuous phase consisting of discrete particle (in this study RBC ghosts particle), the solute is transported to the surface both by convection and by diffusion:

$$\frac{\partial c}{\partial t} + \nabla \cdot (vc) = -\nabla \cdot j \tag{1}$$

where c is the concentration of protein in fluid phase, v is the velocity of the solution and j is the diffusive protein flux through the fluid phase due to Brownian motion. When viewed on microscale structure, the fluid velocity and the concentration fluctuate randomly with position as well as time [Leal, 1977; Zydney and Colton, 1988]. Thus the concentration of protein, the velocity of the fluid, and the flux of protein can be expressed as the sum of a bulk volume-averaged quantity and a fluctuation quantity.

$$c = \bar{c} + c' \tag{2}$$

$$v = \bar{v} + v' \tag{3}$$

$$j = \bar{j} + j' \tag{4}$$

where the overbar represents the bulk quantity and the prime represents the fluctuation. Substituting Eqs. (2)-(4) into Eq. (1) and averaging over the volume gives

$$\frac{\partial(\bar{c})}{\partial t} + \nabla \cdot (\overline{vc}) = -\nabla \cdot \bar{j} - \nabla \cdot \langle v'c' \rangle \tag{5}$$

where by virtue of the definition of the fluctuation terms, the volume average of these terms (e.g $\overline{c'}$, $\overline{v'}$ and $\overline{j'}$) are zero. However, the term $\overline{v'c'}$ is not zero because the local fluctuations in concentration and velocity are not independent of one another [Bird et al., 1960]. The volume average of the product of the concentration and velocity components is expressed in terms of a fluctuating mass flux

$$\langle v'c' \rangle = -D_{ep}\nabla c_t \tag{6}$$

where D_{ep} is effective protein diffusivity due to the particle motions and c_t is the total protein concentration in the suspending solution, hence $\bar{c}$ can be replaced with c_t. In the limit of zero velocity gradient, $\bar{j}$ is the total protein flux through the stationary suspension due to Brownian motion. This flux can be expressed as

$$\bar{j} = -D_s\nabla c_t \tag{7}$$

where D_s is the stationary flow diffusivity. Following the example of Zydney and Colton [1988], Eq. (7), which strictly speaking is valid only in the case of zero velocity gradient, will be used, even though the velocity gradient in this study are non-zero. Substituting Eqs. (6) and (7) into Eq. (5) gives the following equation

$$\frac{\partial c_t}{\partial t} + \nabla \cdot (\bar{v}c_t) = (D_s + D_{ep})\nabla \cdot \nabla c_t \tag{8}$$

The effective protein diffusivity in the suspension is thus given by

$$D_e = D_s + D_{ep} \tag{9}$$

Although both fluid and particle motions are three-dimensional, only the motion perpendicular to the velocity gradient (x direction) was considered in this study. Therefore Eq. (1) is

$$\frac{\partial c_t}{\partial t} + v_x\frac{\partial c_t}{\partial x} = D_e\frac{\partial^2 c_t}{\partial y^2} \tag{10}$$

where the velocity component in the x direction (v_x) [Bird et al., 1960] is

$$v_x = \gamma y\left(1 - \frac{y}{B}\right) \tag{11}$$

where $\gamma = 6Q/B^2W$, B is the thickness of flow chamber, W is the width of flow chamber, and Q is the volumetric flow rate of solution through the rectangular flow chamber. Substituting Eq. (11) into Eq. (10) and assuming steady state gives [Kim and Beissinger, 1993]

$$\gamma y\frac{\partial c_t}{\partial x} = D_e\frac{\partial^2 c_t}{\partial y^2} \tag{12}$$

The following boundary conditions apply:

at $\mathbf{x}=0$,	$\mathbf{c}_t = \mathbf{c}_0$	for all $\mathbf{y}$	(13)
as $\mathbf{y} \to \infty$,	$\mathbf{c}_t = \mathbf{c}_0$	for all $\mathbf{x}$	(14)
at $\mathbf{y}=0$,	$\mathbf{c}_t = 0$	for all $\mathbf{x}>0$	(15)

where c_0 is the feed protein concentration.

Under diffusion-limited conditions, the protein flux to the adsorbing surface from the solution is equal to the measured rate of adsorption onto the surface:

$$\frac{dc_s}{dt} = D_e\left.\frac{\partial c_t}{\partial y}\right|_{\substack{y=0\\x=L}} \tag{16}$$

Applying the Leveque solution to Eqs. (12) to (15) yields

$$\frac{dc_s}{dt} = \frac{1}{\Gamma\left(\frac{4}{3}\right)9^{1/3}}\left(\frac{\gamma}{D_eL}\right)^{1/3} D_e c_o \tag{17}$$

Within the range of fluorescence intensity (F) used in this study, the concentration of adsorbed protein varies linearly with F thus [Cha, 1993]

$$c_s = KF \tag{18}$$

where K is a calibration constant between surface fluorescence intensity (F) versus surface concentration of protein (c_s). Taking the time derivative of Eq. (18) gives

$$\frac{dc_s}{dt}=K\frac{dF}{dt} \tag{19}$$

Substituting this into Eq. (17) and solving for D_e yields

$$D_e=\left\{\frac{K\{dF/dt\}\Gamma(4/3)}{c_0}\right\}^{2/3}\left\{\frac{9L}{\gamma}\right\}^{1/2} \tag{20}$$

The value of dF/dt can be found from the initial portion of fluorescence intensity signal versus time plots which are the results of BSA transport experiment. Since γ, L, c_0 and K are known a priori, the value of D_e can be calculated [Cha, 1993].

The random motion of the particles and fluid elements is constrained by the conservation of volume, which is equivalent to conservation of mass in an incompressible, neutrally-buoyant system. A volume element which is vacated by particle motion must be replaced by fluid [Zydney and Colton, 1988]. Around each particle, the local particle and fluid velocities and concentrations fluctuate randomly with position as well as time. The local instantaneous form of the continuity equation is

$$\frac{\partial}{\partial t}(\rho_i\phi_i)+\nabla\cdot(v_i\rho_i\phi_i)=0 \tag{21}$$

where Brownian motion of the large particles is neglected, v_i is the velocity, ρ_i is the density and ϕ_i is the volume fraction of phase i. The product $\rho_i\phi_i$ is the phase concentration. Zydney and Colton [1988] reported that the following relation [Eq. (22)] between fluid and particle phase was obtained from the continuity equation [Eq. (21)].

$$\rho_p D_p=\rho_f D_f \tag{22}$$

Thus if the fluid and particle densities are equal, the diffusivity of the fluid elements is identical to that of the particles. This equation indicates that the random fluid motion required to fill the volume vacated by the particles must exactly compensate for the random particle motion. In this study the densities of RBC ghosts and the density of PBS solution with FITC-BSA conjugate were measured. The density of the PBS solution with FITC-BSA conjugate (0.075± 0.005 mg/ml) at 20 °C was 1.02 g/cm^3. The density of ghost particles was very close to the density (1.02 g/cm^3) of PBS solution with FITC-BSA conjugate [Cha, 1993]. Therefore the density of the RBC ghosts can be made equal to that of the PBS solution, then Eq. (22) can be simplified to

$$D_p=D_f \tag{23}$$

where D_f is the diffusivity of fluid due to the random motions of the particles. This would be equal to D_{ep} [see Eq. (6)], which is diffusivity of the solute (protein) due to the particle random motions. Thus Eq. (9) can be rearranged as

$$D_p=D_e-D_s \tag{24}$$

where D_e is calculated by Eq. (20) for shear-augmented solute transport (Leveque solution) and experimental results, and D_s is evaluated by following the Meredith and Tobias model which is well valid for the nonconducting spherical particle suspensions [Cha, 1993; Meredith and Tobias, 1968]:

$$\frac{D_s}{D_b}=\frac{8(2-\phi_p)(1-\phi_p)}{(4+\phi_p)(4-\phi_p)} \tag{25}$$

where D_s is the diffusivity of BSA in the stationary and Brownian diffusivity (D_b) of BSA was obtained from results of experiment under homogeneous conditions which is no RBC ghost suspension [Cha and Beissinger, 1996a], and ϕ_p is an RBC ghost volume fraction. Therefore the shear-induced diffusivity of the particles in the suspensions undergoing laminar shear flow can be calculated.

MATERIALS AND METHODS

The experimental methods and materials used were described in the previous papers [Cha and Beissinger, 1996a, b]. In this paper, the experimental approach will be briefly described. The rectangular flow chamber was used for making continuous measurements by

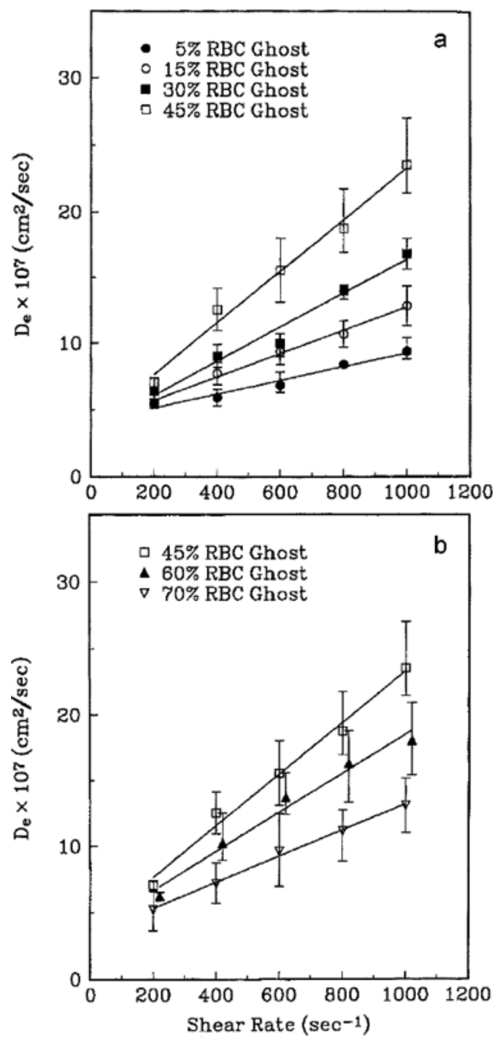

Fig. 1. (a) The effective diffusivity of BSA at RBC ghost volume fraction of 0.05 to 0.45 as a function of shear rate; Data are represented by the mean along with ±1 standard deviation. (b) The effective diffusivity of BSA at RBC ghost volume fraction of 0.45 to 0.7 as a function of shear rate; Data are represented by the mean along with ±1 standard deviation.

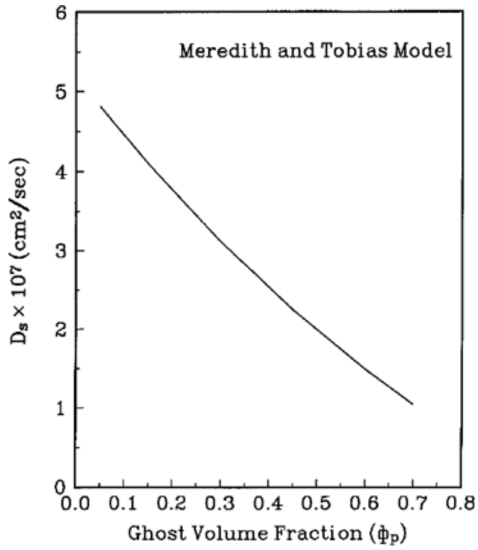

Fig. 2. Diffusivity of BSA in stationary suspension as a function of particle volume fraction.

total internal reflection (TIRF) method under diffusion-limited conditions. BSA was conjugated by fluorescein isothiocyanate (FITC) to use TIRF method. The concentration of conjugated BSA-FITC in 0.175 M PBS solution was 0.075±0.005 mg/ml. RBC ghost which is a membrane of RBC without hemoglobin, were used as a suspended material. Preparation method of RBC ghost was reported at previously [Cha and Beissinger, 1996b]. The effective diffusivity of BSA in RBC ghost suspensions range of 0.05-0.7 was measured over the shear rate range of 200-1,000 s^{-1} and pH 7.4, at 20 °C.

RESULTS AND DISCUSSION

Fig. 1a and 1b showed the effective diffusivity (D_e) of BSA in the RBC ghost suspensions as a function of shear rate which was calculated by the Eq. (20) and experimental results [Cha and Beissinger, 1996b]. The effective diffusivity of BSA is a function of a shear rate and a volume fraction of RBC ghost particle.

The diffusivity of BSA in stationary condition was calculated by Eq. (25) using D_b is 5.19×10^{-7} cm²/sec [Cha and Beissinger, 1996a]. Fig. 2 shows the results for the diffusivity of BSA in stationary (D_s). Therefore the shear-induced particle diffusivity (D_p) can be evaluated by Eq. (24) and above results, and plotted as a function of shear rate in Fig. 3a and 3b. As the shear rate increased, D_p was increased. Both the rotational and the translational motions are contributed to the particle diffusion. Each particle rotates with an angular velocity approximately half the mean shear rate of the fluid [Jefferey, 1922]. This motion would be produced a rotational flow in the nearby fluid which exerts drag forces on neighboring particles. Each particle overtakes and passes particles on slower moving streamlines. During these passing interactions or collisions, the particles will be transiently displaced, i.e. net lateral migrations occur [Eckstein et al., 1977]. For any given particle, the rate of interactions with other particles is proportional to the number of particles (i.e. the particle volume fraction) and shear rate [Leighton and Acrivos, 1987]. Therefore as the shear rate increased, the contribution of the rotational and the translational motion to the particle diffusion would be increased. Fig. 4 showed particle diffusivity as a function of ghost volume fractions. The particle diffusivity had a maximum value at ghost fraction of 0.45. It seemed that there are three effects of rotation, translation, and deformation of the ghost particle (deformable particle) on the particle migration. As the volume fraction increases from 0 to 0.2, the rotational and the translational motion of RBC ghost particle were expected to increase. At ϕ_p>0.3, the continuous and remarkable deformation of RBC ghost particle was occurred, and which was increased with increasing volume fraction [Goldsmith and Marlow, 1979]. Therefore it is expected that the rotational motion of the particles decrease because of the deformation of the particles and the presence of the particle in the neighbor. Although there was some deformation of the particles, total contributions of the rotation, translation, and deformation would increase until the particle diffusivity reached a maximum value at ghost volume fraction of 0.45, but the increment of the contributions would

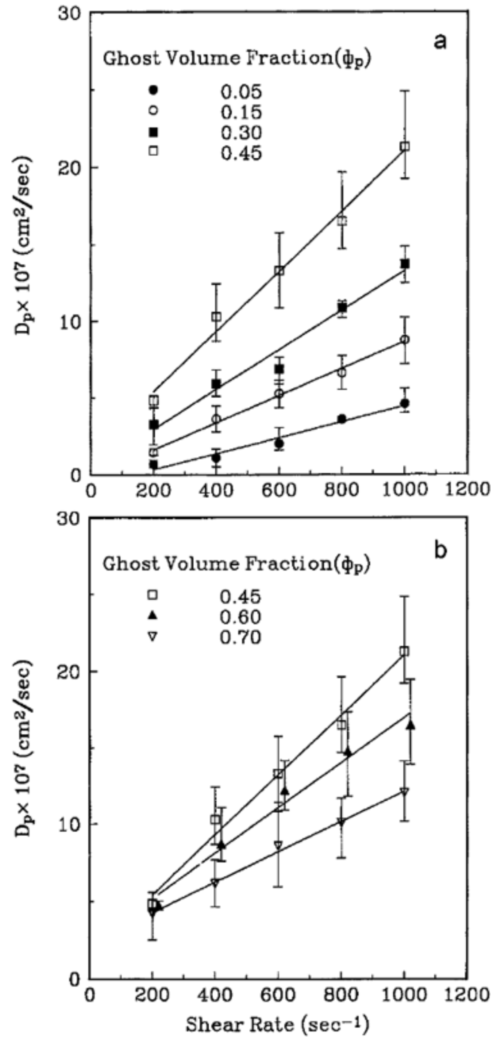

Fig. 3. (a) The shear-induced particle diffusivity at RBC ghost volume fraction of 0.05 to 0.45 as a function of shear rate; Data are represented by the mean along with ±1 standard deviation. (b) The shear-induced particle diffusivity at RBC ghost volume fraction of 0.45 to 0.7 as a function of shear rate; Data are represented by the mean along with ±1 standard deviation.

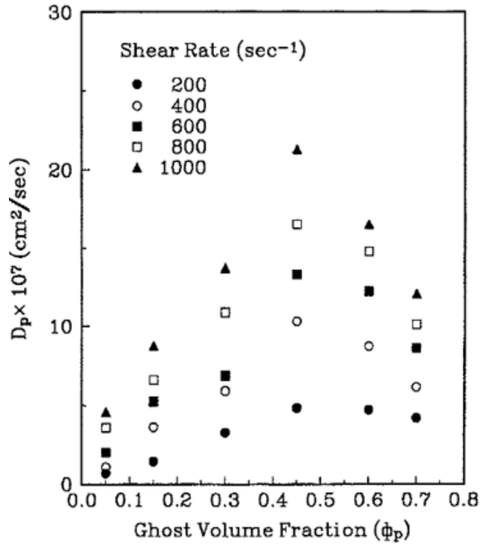

Fig. 4. The shear-induced particle diffusivity vs. ghost volume fraction; Data are represented by the mean value.

be decreased. As the volume fraction increased above 0.45, the rotational motion decreased until the ghost particles no longer rotated as a whole. As the volume fraction of the particles increased, the deformation of the particles increased. Therefore total contribution of rotation, translation, and deformation of RBC ghost particle would be decreased.

Eckstein et al. [1977] suggested that the dimensionless particle diffusivity ($D_p/a^2\gamma$) is a function of the following dimensionless group:

$$\frac{D_p}{a^2\gamma}=f\left\{\phi, \frac{a}{w}, \frac{a}{y}, \frac{\rho_f a^2\gamma}{\mu}, \frac{a\partial^2 v_y/\partial y^2}{\gamma}, \frac{g(\rho_p-\rho_f)a}{\mu\gamma}\right\} \tag{26}$$

where a/w and a/y are the geometric parameters, the term $\rho_f a^2\gamma/\mu$ is the Reynolds number of the flow. The group $(a\partial^2 v_y/\partial y^2)/\gamma$ is a measure of the fractional change in shear rate over a distance of one particle radius, and the last dimensionless term $g(\rho_p-\rho_f)a/\mu\gamma$, is a measure of the ratio of the settling speed produced by the unbalanced body force to the speed, and γa is characterizing the local shear flow relative to a particle. If the problem would be that of an infinite, inertia-free, two-dimensional, neutrally buoyant, linear shear flow, with the particle migrations wholly governed by shear-induced particle diffusion, Eq. (26) would reduced to

$$\frac{D_p}{a^2\gamma}=f\{\phi_p\} \tag{27}$$

Therefore the shear-induced particle diffusivity data for suspensions of RBC ghost particles were fitted to the form [Zydney and Colton, 1988]

$$\frac{D_p}{a^2\gamma}=m\phi_p(1-\phi_p)^n \tag{28}$$

where m and n are constants which were calculated by curve fitting. Experimental results for the dimensionless shear-induced particle diffusivity ($D_p/a^2\gamma$) and the result of best fit of Eq. (28) were plotted in Fig. 5 as a function of particle volume fraction (ϕ_p). The best fit constants with standard deviations were m=0.07982±0.007, and

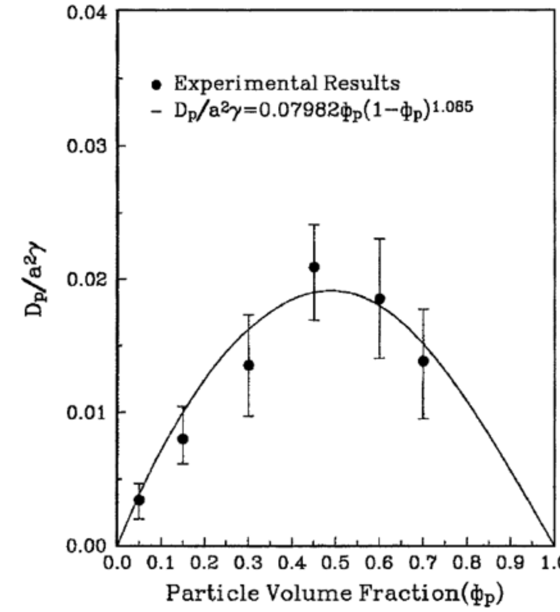

Fig. 5. Dimensionless shear-induced particle diffusivity vs. particle volume fraction with the best fit to the diffusivity data, Eq. (29); Data are represented by the mean along with ±1 standard deviation.

n=1.085±0.1. These values when inserted into Eq. (28) yield the following correlation:

$$\frac{D_p}{a^2\gamma}=0.07982\pm0.007\phi_p(1-\phi_p)^{1.085\pm0.1} \tag{29}$$

Zydney and Colton [1998], and Kim [1990] also used this empirical parameters. Zydney and Colton [1998] obtained the best fit values and standard deviation: m=0.15±0.03 and n=0.8±0.3. Kim [1990] presented the constants of m=0.17 and n=1.1. This empirical equa-

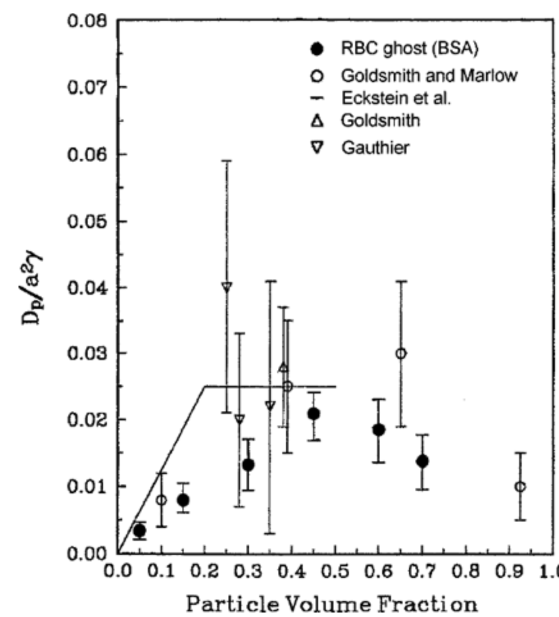

Fig. 6. Dimensionless shear-induced particle diffusivity comparing with literature values.

Table 1. The correlated dimensionless shear-induced particle value by Eq. (32) with experimental results

ϕ_p	$(D_p/a^2\gamma)_{correlated}$	$(D_p/a^2\gamma)_{exp.}$
0.05	0.0038±0.0002	0.0034±0.0007
0.15	0.0079±0.0002	0.0077±0.0006
0.30	0.0123±0.0003	0.0128±0.0015
0.45	0.0199±0.0006	0.0209±0.0017
0.60	0.0167±0.0006	0.0185±0.0023
0.70	0.0119±0.0006	0.0138±0.0029

tion predicts that the dimensionless shear-induced particle diffusivity for deformable particles reaches a maximum value at volume fraction of about 0.45 to 0.5. The values of $D_p/a^2\gamma$ were compared with other literature values in Fig. 6. These literature values represent data for deformable particles: RBC ghost [Kim and Beissinger, 1993], liquid drops [Gauthier et al., 1972], and red blood cells [Goldsmith and Marlow, 1979; Goldsmith, 1971]. The solid line represents the correlations developed by Eckstein et al. [1977]

$$\frac{D_p}{a^2\gamma} = 0.025\left(\frac{\phi_p}{0.2}\right) \qquad 0<\phi_p<0.2 \tag{30}$$

$$\frac{D_p}{a^2\gamma} = 0.025 \qquad 0.2<\phi_p<0.5 \tag{31}$$

Experimental results in this study were in good agreement with other literature values.

The particle diffusion produces augmented diffusion of solute in suspensions. The augmented diffusion experimental results and analysis were reported previously [Cha and Beissinger, 1996b]. Augmentation in solute transport can be related with shear-induced particle diffusivity. The relation between augmentation (A) and shear-induced particle diffusivity was:

$$A = \frac{D_e - D_s}{D_s} = \frac{D_p}{D_s} = \left(\frac{D_p}{a^2\gamma}\right)Pe \tag{32}$$

where $Pe=a^2\gamma/D_s$. Augmentation of BSA in RBC ghost suspensions was proportional to the Pe [Cha and Beissinger, 1996b]. By the least square method, proportional constant, which was to be dimensionless shear-induced particle diffusivity by Eq. (32) was calculated and summarized in Table 1. The correlated dimensionless shear-induced particle diffusivity is very close to experimental results.

SUMMARY

The shear-induced particle diffusivity of RBC ghost in sheared suspensions under laminar flow condition can be evaluated based on experimental results and flow model. The shear-induced particle diffusivity was calculated based on the relation which the shear-induced particle diffusivity (D_p) equals to the effective diffusivity (D_e) of BSA in the RBC ghost suspension minus the diffusivity (D_s) of BSA in the stationary condition. The effective diffusivities (D_e) of BSA under diffusion-limited condition were investigated using the TIRF method at a shear rate range of 200-1,000 s^{-1} and RBC ghost volume fraction range of 0.05-0.7. The diffusivity (D_s) of BSA in the stationary condition was determined by Meredith and Tobias model.

The range of shear-induced particle diffusivities was 0.35×10^{-7} to 21.2×10^{-7} cm^2/s. The shear-induced particle diffusivity of RBC ghost was a function of shear rate and particle volume fraction and it reached a maximum value at particle volume fraction of 0.45. The rotational motion, translational motion and deformation of the RBC ghost particle contributed to the particle diffusion.

The dimensionless particle diffusivity ($D_p/a^2\gamma$) was described by

$$\frac{D_p}{a^2\gamma} = 0.07982\phi_p(1-\phi_p)^{1.085}$$

and $D_p/a^2\gamma$ was in good agreement with literature values.

ACKNOWLEDGEMENT

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NOMENCLATURE

a	: radius of RBC ghost particle [cm]
B	: thickness of flow chamber [cm]
c	: concentration of protein [mg/ml]
c_0	: feed protein concentration [mg/ml]
c_t	: total protein concentration [mg/ml]
D_e	: effective diffusivity of solute [cm^2/s]
D_{ep}	: effective diffusivity of protein due to the particle motion [cm^2/s]
D_p	: shear-induced particle diffusivity [cm^2/s]
D_s	: stationary diffusivity of solute [cm^2/s]
g	: gravitational acceleration [cm^2/s]
j	: diffusive protein flux [mg/cm·s]
K	: calibration constant
L	: distance from the entrance to detection point [cm]
m, n	: constant
Q	: volumetric flow rate of solution [cm^3/s]
t	: time
v	: velocity [cm/s]
W	: width of rectangular flow chamber [cm]
x	: axial distance in the direction flow [cm]
y	: distance from adsorbing wall [cm]

Greek Letters

Γ	: Gamma function
γ	: shear rate [s^{-1}]
ρ_i	: density of phase i [g/cm^3]
ρ_p	: density of particle [g/cm^3]
ρ_f	: density of fluid [g/cm^3]
ϕ_p	: ghost volume fraction

REFERENCES

Bird, R. B., Stewart, W. E. and Lightfoot, E. N., "Transport Phenomena," John Wiley & Sons Inc. (1960).

Cha, W., Ph.D. Dissertation, "Red Blood Cell-Augmented Mass Transport of Albumin in Sheared Suspensions to Surfaces," Illinois Institute of Technology (1993).

Cha, W. and Beissinger, R. L., "Macromolecular Mass Transport to a Surface: Effects of Shear Rate, pH and Ionic Strength," *J. Colloid Interface Sci.*, **177**, 666 (1996a).

Cha, W. and Beissinger, R. L., "Augmented Mass Transport of Macromolecules in Sheared Suspensions to Surfaces; B. Bovine Serum Albumin," *J. Colloid Interface Sci.*, **178**, 1 (1996b).

Chin, B. D. and Park, O. O., "Electrorheological Responses of Particulate Suspensions and Emulsions in a Small-Strain Dynamic Shear Flow: Viscoelasticity and Yielding Phenomena," *Korean J. Chem. Eng.*, **18**, 54 (2001).

Eckstein, E. C., Bailey, D. G. and Shapiro, A. H., "Self-Diffusion of Particles in Shear Flow of a Suspensions," *J. Fluid Mech.*, **79**, 191 (1977).

Gauthier, F. J., Goldsmith, H. L. and Mason, S. G., "Flow of Suspensions Through Tubes, X. Liquid Drops as Models of Erythrocytes," *Biorheology*, **9**, 205 (1972).

Goldsmith, H. L., "Red Cell Motions and Wall Interactions in Tube Flow," *Fed. Proc.*, **30**, 1578 (1971).

Goldsmith, H. L. and Marlow, J. C., "Flow Behavior of Erythrocytes, II. Particle Motions in Concentrated Suspension of Ghost Cells," *J. Colloid Interface Sci.*, **71**, 383 (1979).

Jefferey, G. B., "The Motion of Ellipsoidal Particle Immersed in a Viscous Fluid," *Proc. Roy. Soc.*, **A102**, 161 (1922).

Kim, D., Ph.D. Dissertation, "Augmentation of Macromolecular Mass Transport in Sheared Suspensions: The Effective Diffusivity of Gamma Globulin in Red Blood Cell Ghosts Suspensions," Illinois Institute of Technology (1990).

Kim, D. and Beissinger, R. L., "Mass Transport of Macromolecules in Solution to Surfaces," *J. Colloid Interface Sci.*, **159**, 9 (1993).

Leal, L. G., "Macroscopic Transport Properties of a Sheared Suspension," *J. Colloid Interface Sci.*, **58**, 296 (1977).

Leighton, D. and Acrivos, A., "The Shear-Induced Migration of Particles in Concentrated Suspensions," *J. Fluid Mech.*, **181**, 415 (1987).

Meredith, R. E. and Tobias, C. W., "Conductives in Emulsions," *J. Electrochemical Soc.*, **108**, 286 (1968).

Yim, S. S., "A Theoretical and Experimental Study on Cake Filtration with Sedimentation," *Korea J. Chem. Eng.*, **16**, 308 (1999).

Zydney, A. L. and Colton, C. K., "Augmented Solute Transport in the Shear Flow of a Concentrated Suspension," *Physicochemical Hydrodynamics*, **10**, 77 (1988).